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Oblique Stagnation Point Flow of Maxwell Trihybrid Nano-Material Over a Stretching Cylinder

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ABSTRACT: The goal of the basic work is to, better understand how a Maxwell nanofluid flows in the stretching cylinder with active and passive controlled nanoparticles and study about oblique stagnation point blood flow of Maxwell nanomaterials over a stretching cylinder. Three different types of nanoparticles $Cu + Al_2O_3 + TiO_2$ are considered with blood used as base fluid. This is accomplished by deriving the fluid flow and heat transfer governing equations from the conservation laws of mass, momentum, concentration and energy. Following that, the resulting partial differential equations are numerically solved using the proper techniques and boundary conditions. Similarity, transformation are applied to reduce the governing partial differential equations into a system of nonlinear ordinary differential equations, which are solved by numerically by shooting method coupled with Runge-Kutta method BVP4C. The effects of Prandtl number, Schmidt number, Thermophoresis parameter, Velocity ratio parameter, types of nanoparticles, nanoparticles concentration at the surface of cylinder, radius of cylinder and length of cylinder, effect flow behavior with active and passive nanoparticles on stretching cylinder are graphically illustrated and discussed. The main objective is to examined the oblique stagnation point flow of incompressible Maxwell nanofluid past a stretching cylinder with regulated active and passive nanoparticle concentration conditions.

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I. INTRODUCTION

A fluid is a substance that continually deforms (flows) under an applied shear stress regardless of the magnitude of the applied stress. Fluid may be static or dynamic. The fluid at rest is called static fluid while the fluid at motion is known as dynamic fluid. A fluid called a nanofluid contains nanoparticles, which are microscopic fibres or particles. Choi used the term "nanofluid" in 1995 to describe artificial colloids made up of nanoparticles dispersed in a base fluid. Nanoparticles are frequently made of oxides like alumina, silica, and titania as well as metals like copper and gold. Nanofluids have also been created using carbon nanotubes and diamond nanoparticles. Water and organic fluids like blood, ethanol and ethyleneglycol are common base fluids, and the volumetric proportion of nanoparticles in the base fluid is typically less than 5%. These fluid mixtures were discovered to have exceptional qualities that make them potentially useful in many heat transfer applications, such as microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines, engine cooling. When uniformly dispersed and suspended in stable suspension in traditional fluids, a very tiny number of substantially greater thermal conductivity nanometer-sized particles can dramatically improve the thermo-physical properties of the base fluid. Fluid is also categorized into two forms that are Newtonian and Non-Newtonian. Numerous industrial fluids have non-Newtonian flow properties. A fluid that has constant viscosity, zero shear rate, and zero shear stress is referred to as a Newtonian fluid and Newtonian fluid obey the newtons law. While the Non-Newtonian fluids do not obey Newton's law, their viscosity varies and is dependent on the shear rate. Numerous industrial fluids have N0n-Newtonian flow properties. Shear stress and rate of shear strain have a nonlinear con-

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nection in Non-Newtonian fluids compared to Newtonian fluids where shear stress and rate of shear strain are exactly proportional. Example of Newtonian fluids are air,water,mineral oil, alcohol. Blood, paint, toothpaste, and starch solutions are a few examples of non-Newtonian fluids. Hiemenz first examined stagnation point flow in 1911. He looked at a stationary plate's two-dimensional stagnation point flow. Engineering and industry have practical uses for the movement and heat transmission across the stretched surfaces at the stagnation point zone, such as cooling metallic plates, electrical gadgets and fan-equipped nuclear reactors, continuous costing, glass and plastic sketching, and crystal puffing, hot rolling food, spinning metal, producing paper, drawing wire, etc. Industrial working fluids are mixtures of many substances including water, oil, and other chain molecules.

A Maxwell fluid is defined as the viscoelastic substance having the properties which is elasticity and viscosity. The kind of fluid which is purely viscous and elastic and are jointed in series can be expressed as Maxwell in 1867 this model is proposed by James Clerk Maxwell. Viscosity is a measure of a fluid's thickness and resistance. Viscous fluids, such as motor oil, ketchup, toothpaste and shampoo are regarded as being highly gloppy fluids. Fluids will always be important, and how best to use them will depend on how they are categorized.Non-Newtonian fluids are typically categorized as liquids including oil, blood, custard, and juice that experience nonlinear connections with shear stress. They are further split into numerous categories for research purposes based on characteristics that have been premeditatedly examined and studied. Oldroyd-B fluid was studied by Jamil et al.[1], Swati et al.[2] examined the impact of transpiration on Casson fluid,Jeffery fluid by Hayat et al.[3], Micropolar fluid flow by Sheikholeslami et al.[4], and a third-grade fluid by Ellahi et al.[5] compensated for heat transmission. In the industrial sector, viscoelastic fluids with extremely elastic property impacts, such as polymeric solution, glycerin, and crude oil, are accounted for by the upper-convected Maxwell (UCM) model. The utility of the UCM fluid property in relation to different physical quantities, geometries, stretchable and non-stretchable sheets has been studied by a number of writers. A Maxwell fluid research by Hsiao[6] showed that a higher visco-elastic number results in a larger rate of heat transfer.

Flow around cylinders is a challenge in engineering design. The flow around cylinders is analogous to the flow around chimneys, buildings, bridges, and heat exchanger pipes in real life. Uncontrolled flow can result in vortex shedding (unsteady wake) behind the body, which will produce strong fluid forces and flow-induced vibrations. Methods of flow control can lessen the flow's forces, separation, wake, and unsteadiness. Effective flow management can reduce induced body vibration, save energy, and improve propulsion efficiency.The study of forces acting on an item moving through a fluid is known as fluid mechanics. The three main fluid forces are lift, drag, and buoyancy.T he drag force acts in the opposite direction of the relative flow velocity.In opposition to the relative flow velocity, the lift force operates. Pressure and friction, which make up these two forces, are what this subject is mostly interested in. Active and passive control methods were used to categorize flow control strategies by Malekzadeh and Sohankar[7]. Active control techniques use outside energy to regulate the flow. A jet blower is one illustration of this technique. Passive control techniques alter the body's form or affix additional devices to control the flow. Control plates and roughness components are a couple of examples of passive control techniques. Passive control methods are less complicated, less difficult, and more affordable than active control approaches. Many scientists investigated various passive control techniques to lessen the fluid forces acting on the body during flow.Upstream flat control plate was explored by Ali et al.[8] Slit and shell simulations were done on the body in the flow by Baek and Karniadakis[9].

We used here tri-hybrid nanoparticles $(\mathbf{Cu}, \mathrm{TiO_2}, \mathrm{Al_2O_3})$ with base flood blood. Here we check the effect of these three nanoparticles on flow of stretching cylinder. and we conclude our remarks and results also discuss by graphically. Blood is a multi-component non-Newtonian fluid that is primarily used in natural science issues. It is made up of platelets, red and white blood cells, plasma, and other substances. While the foundation fluid in our present model is "pure blood," it is important to keep in mind that, from a medical standpoint, blood viscosity is not really constant and can be affected by temperature, hematocrit, stress, and other factors in addition to vessel diameter. It has also been stated in multiple studies that the assumption of Newtonian blood performance is appropriate for high shear rate motion, such as flow through blood arteries.Generally speaking, blood thickness reduces as blood circulation increases, increasing blood flow velocity and lowering clotting factors.

II. MATHEMATICAL FORMULATION

Consider a steady, two-dimensional, oblique stagnation point blood flow of Mawxell nanomaterials over a stretching cylinder in the current investigation. A

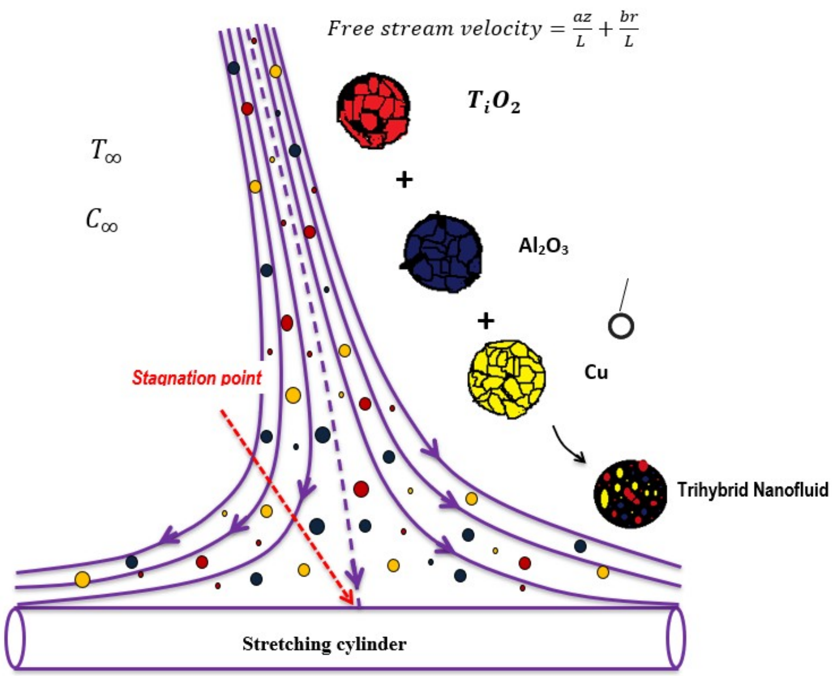

FIG. 1: Geometry of stretching cylindrical.

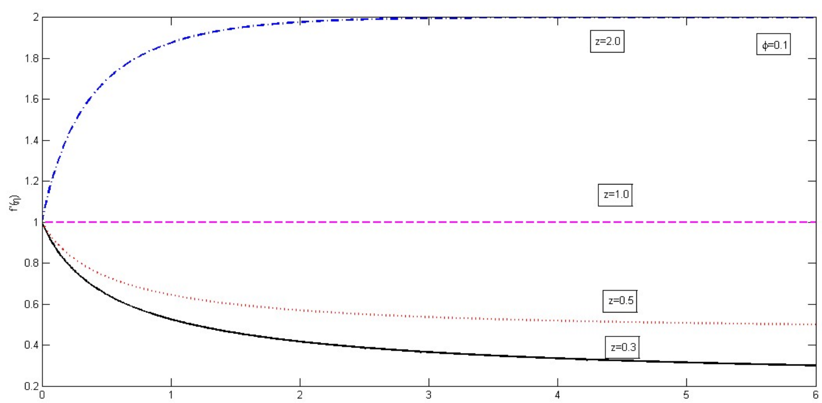

FIG. 2: $f'(\eta)$ in opposition to $\frac{a}{c}$ for ϕ_1.

stretching cylinder of constant radius R is imaged obliquely by a steady flow of in-compressible Maxwellian nanomaterial. Figure 1 displays the coordinate system and a physical drawing of the issue. It demonstrates that the blood flow of three trihybrid nanoparticles, $Cu + Al_2O_3 + TiO_2$, is caused by Mawxell nanomaterials stretched along a cylinder. With velocity $U_w = \frac{cz}{L}$, where c is the reference velocity and L is the particular length, the cylinder is quickly extending in the axial direction. Below is a description of the current problem's leading equations:

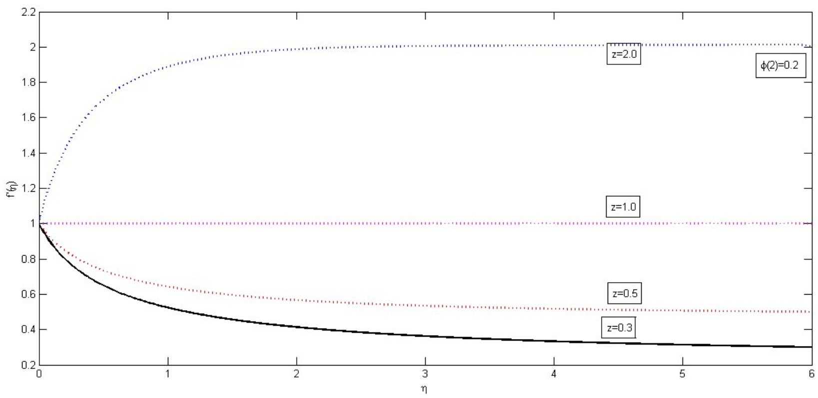

FIG. 3: $f'(\eta)$ in opposition to $\frac{a}{c}$ for ϕ_2.

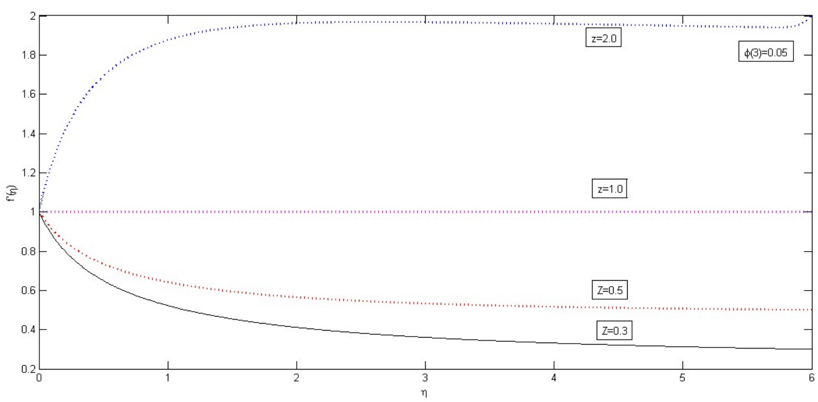

FIG. 4: $f'(\eta)$ in opposition to $\frac{a}{c}$ for ϕ_3.

In the absence of body force, the governing equations and near the boundary layer's edge.

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\begin{aligned} &u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} + \lambda_1\left(w^2\frac{\partial^2 w}{\partial^2 z} + u^2\frac{\partial^2 w}{\partial^2 r} + 2uw\frac{\partial^2 w}{\partial r \partial z}\right) \\ &-\left(\frac{b}{c}\right)A + \nu_{\text{thnf}}\left(+\frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial^2 r}\right) + \left(\frac{a}{c}\right)^2 z \end{aligned} \tag{2}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k_{thnf}}{(\rho C)_{thnf}}\left(\frac{\partial^2 T}{\partial^2 r}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+ \tau_1\left[D_B\frac{\partial T}{\partial r}\frac{\partial C}{\partial r}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial r}\right)^2\right] \tag{3}$$

$$u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=D_B\left(\frac{\partial^2 C}{\partial^2 r}+\frac{1}{r}\frac{\partial C}{\partial r}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial^2 r}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{4}$$

The end point equations are as follows:

$$\left.\begin{array}{r} w=\frac{cz}{L},\quad u=0,\quad T=T_w, \\ C=C_w(\text{ Active controler })\quad \text{at } r=R, \\ D_B\frac{\partial C}{\partial r}+\frac{D_T}{T_\infty}\frac{\partial C}{\partial r}=0.(\text{ Passive controler }) \text{ at }\quad r\to\infty \\ w=\frac{az}{L}+\frac{br}{L}, T=T_w, C=C_w. \end{array}\right\} \tag{5}$$

Now to renovate the above system of partial differential equations into non linear set of dimensionless form of differential equations consider following variables:

$$\left.\begin{array}{ll} \psi=R\sqrt{\frac{c\nu}{L}}(zf(\eta)+g(\eta)), & \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, & \eta=\left(r^2-R^2\right)\sqrt{\frac{c}{\nu L}} \end{array}\right\} \tag{6}$$

After using the dimensionless variable and extended parameters comparing the coefficient of z^0 and z^1 we have:

$$\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)f'''+2\sqrt{\frac{A_1}{A_2}}\gamma f''+ff''-f'^2+\beta_1\left[-f^2f'''+2ff'f'' -\frac{\sqrt{\frac{A_1}{A_2}}\gamma}{\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)}f^2f''\right]=-\left(\frac{a}{c}\right)^2 \tag{7}$$

$$\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)g''+2\sqrt{\frac{A_1}{A_2}}\gamma g''+fg''-f'g'+\beta_1\left[-f^2g'''+2fg'f'' -\frac{\sqrt{\frac{A_1}{A_2}}\gamma}{\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)}f^2g''\right]=A\lambda \tag{8}$$

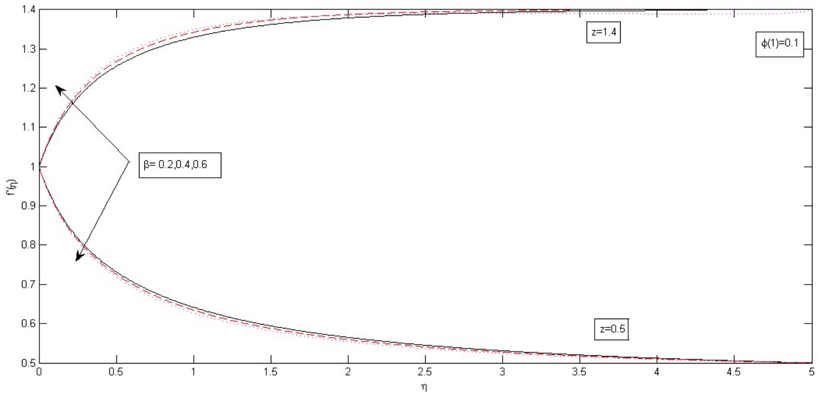

FIG. 5: $f'(\eta)$ in opposition to β_1 for ϕ_1.

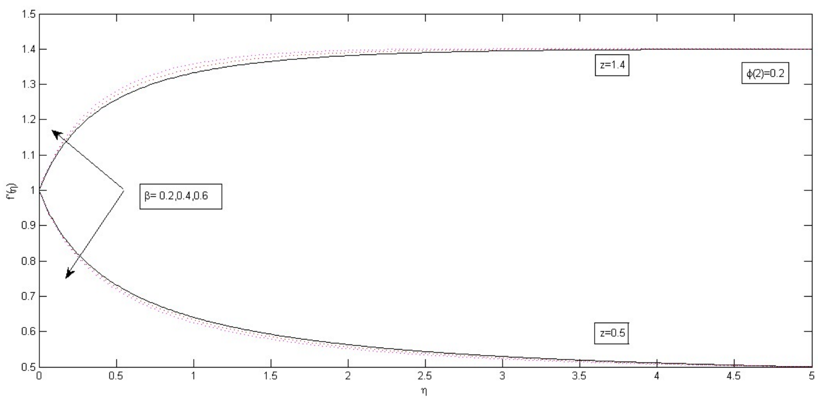

FIG. 6: $f'(\eta)$ in opposition to β_1 for ϕ_2.

$$\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)\theta'''+2\sqrt{\frac{A_1}{A_2}}\gamma\theta'+\frac{A_1A_3}{A_2A_4}\,\mathrm{Prf}\,\theta'+\frac{A_1A_3}{A_2A_4}\,\mathrm{Pr}\left[\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)\theta'^2 +Nb\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)\theta'\phi'\right]=0 \tag{9}$$

$$\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)\phi'''+2\sqrt{\frac{A_1}{A_2}}\gamma\phi'+\frac{A_4}{A_3}Scf\phi'+\frac{Nt}{Nb}\left[\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)\theta''+2\sqrt{\frac{A_1}{A_2}}\gamma\theta\right]=0 \tag{10}$$

Subject to the following boundary condition:

$$\left.\begin{array}{r} f(0)=0, g(0)=0, \quad f'(0)=1, \quad g'(0)=0, \quad \theta(0)=1 \\ \Phi(0)=0, \quad (\text{ Active controler }) \\ \phi'(0)+\frac{Nt}{Nb}\theta\prime(0)=0 \quad (\text{ Pasive controler }) \\ g''(\infty)=\lambda, \quad f'(\infty)=\frac{a}{c}, \ \theta(\infty)=0, \quad \phi(\infty)=0 \end{array}\right\} \tag{11}$$

In above equations curvature of cylinder, Maxwell fluid parameter, Prandtl number, thermophorsesis parameter, Brownian motion parameter, Schmidt number are:

$$\left.\begin{array}{l} \gamma=\sqrt{\frac{\nu L}{cR^2}}, \quad B_1=\frac{\lambda_1 c}{L}, \quad \Pr=\frac{\nu}{\alpha}, \\ Nt=\frac{\tau_1}{\nu}\frac{D_T}{T_\infty}(T_w-T_\infty), \quad Nb=\frac{\tau_1}{\nu}D_b(C_w-C_\infty). \end{array}\right\} \tag{12}$$

let $\quad h(\eta)=\lambda g' \quad$ then equation (8) becomes:

$$\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)h''+2\sqrt{\frac{A_1}{A_2}}\gamma h'+fh'-f'+\beta_1\left[-f^2h''+2fh'f''-\frac{\sqrt{\frac{A_1}{A_2}}\gamma}{\left(1+2\sqrt{\frac{A_1}{A_2}}\gamma\eta\right)}f^2h'\right]=A \tag{13}$$

Subject to the boundary conditions:

$$h(0)=0, \quad h(\infty)=1 \tag{14}$$

III. MAIN RESULTS

In this paper an issue of two-dimensional stable and in compressible Maxwell nanomaterials near a stagnation point is introduced. This problem involves both active and passive control of the nanoparticles. Through tabular data, the current results are compared to the body of literature under various suppositions. The blood is taken as base fluid is taken with the $Cu + Al_2O_3 + TiO_2$ as trihybrid particles. Table I shows a good agreement of numerical results with the previously published observations in a limiting case. In Figs. 2-4, the relationship between the axial velocity $f'(\eta)$ and the increasing values of the velocity ratio parameter $\frac{a}{c}$ is depicted for the values $\frac{a}{c}=0.3, 0.4, , 1.0, 2.0$. In these Fig's it is examined that axial velocity $f'(\eta)$ decrease at the value of velocity ratio parameter $\frac{a}{c}=0.3$ and axial velocity f' increase rapidly at $\frac{a}{c}=2.0$ also for the ϕ_1 axial velocity increase more rapidly as compared to ϕ_2, ϕ_3 In more the thickness of boundary layer reduces by increasing velocity ratio parameter $\frac{a}{c}$. An increase in the concentration of nanoparticles typically enhances heat transfer efficiency due to improved thermal conductivity. However, excessive nanoparticle concentration may lead to higher viscosity, which can resist the fluid motion. Due to which the velocity of the fluid becomes lower.

TABLE I: Comparing of the numerical values in limiting case.

values of r	$f''(0)$		
	Present	Ref.[10]	Ref.[11]
0	2.6083	2.608	2.603
0.1	2.5714	2.571	2.571
0.2	2.465	2.46	2.455
0.3	2.288	2.28	2.275

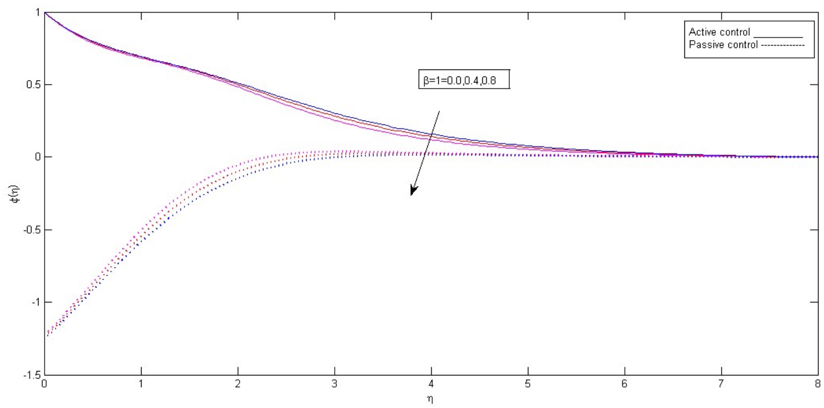

FIG. 7: $\phi'(\eta)$ in opposition to β_1.

In Fig 5, the relationship between axial velocity and increasing value of Maxwell fluid parameter $\beta_1 = 0.2, 0.4, 0.6$ is plotted. For various values of $\frac{a}{c}$, Fig. 5 shows the effects of non-Newtonian Maxwell fluid parameter β_1 on the axial velocity profile. The axial velocity increases by increasing the non-Newtonian behavior which corresponds to the β_1 for $\frac{a}{c} > 1$ and axial velocity reduces by increasing β_1 when $\frac{a}{c} < 1$. For velocity ratio parameter $\frac{a}{c} = 0.5$ and increasing value of Maxwell fluid parameter β_1 axial velocity decrease and velocity ratio parameter $\frac{a}{c} = 1.4$ and increasing value of Maxwell fluid parameter β_1 axial velocity increase.

For various values of a/c, Fig. 6 shows the effects of non-Newtonian Maxwell fluid parameter β_1 on the tangential velocity. As we change the nature of fluid from Newtonian to non-Newtonian fluid the tangential velocity $g'(\eta)$ decreases. The temperature profile and nanoparticle concentration profile responses to increasing values of the nonNewtonian fluid parameter β_1 are examined qualitatively. In Fig. 7 concentration profile is plotted for both active and passive nanoparticles. Passive nanoparticles can be used to manage the increasing non-Newtonian behavior, which lowers the temperature profile, concentration profile, and lowering magnitude. For the increasing value of Maxwell fluid parameter $\beta_1 = 0.0, 0.3, 06$ temperature profile decrease.This indicates that an increase in viscoelasticity reduces internal kinetic energy, which in turn reduces the rate of heat transmission and, ultimately, the thermal profile. As the Maxwell fluid model is a viscoelastic fluid that accounts for the elastic effects of the fluid. The parameter β_1 represents the relaxation time of the fluid. Higher values indicate stronger viscoelastic behavior, meaning the fluid retains the effects of past deformations for a longer duration.

IV. CONCLUSION

This paper introduces the topic of two-dimensional stable and in compressible Maxwell nanomaterial approaching a stagnation point, imagined obliquely across a stretching cylinder with nanoparticles under active and passive control. Using the shooting method or Rk4 approach, the modeled boundary layer partial differential equations are numerically solved. Using tabular data, the current findings are compared to the body of literature under various presumptions. The effects of intriguing newly discovered parameters on many aspects of oblique stagnation point flow are illustrated and tabulated in accordance with active and passive nanoparticle control. We conclude our remarks by using trihybrid nanoparticles with base fluid blood and discuss what will be effects. The main conclusions are outlined below.

- Contrary to what is written in the literature, the tangential velocity $g'(\eta)$ drops when the fluid's nature changes from Newtonian to non-Newtonian.
- Nanoparticle concentration and temperature distribution are reduced by the elasticity of non-Newtonian fluid.
- Because of the huge cylinder curvature, increasing thermophoresis dispersion lowers nanoparticle concentration, which goes against existing research.
- A rise in the Schmidt number causes the concentration of nanoparticles to increase, allowing for passive nanoparticle control.
- While the curvature of the circular cylinder raises $h'(0)$, the increase in elasticity decreases both $f''(0)$ and

$h'(0)$.
- The Sherwood number decreases as thermophoresis diffusion occurs for both active and passive nanoparticle control.
- By increasing Brownian motion, the Maxwell fluid's Sherwood number is smaller than that of the Newtonian fluid.
- As the heat and mass diffusivities rise, the Nusselts number decreases.
- By increasing the fluid's elasticity, the obliqueness in streamlines can be managed.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no conflict of interest. Future work directions: While our study specifically focuses on blood as a base fluid due to its relevance in biomedical applications, we acknowledge that exploring other base fluids (e.g., water, ethylene glycol) could provide a broader perspective on the impact of trihybrid nanoparticles. Further, the experimental validation would further strengthen the study. However, the current work primarily focuses on theoretical and numerical analysis. Experimental investigations would require extensive lab resources and are beyond the present scope. Nonetheless, we highlight the need for such validation in future studies. The inclusion of magnetic effects (MHD flow) is indeed an interesting direction, especially considering biomedical applications such as targeted drug delivery and hyperthermia treatments. While this study does not incorporate an applied magnetic field.

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